Estimating Tail Factors
Using an Exponential Decay Model

ARCHER ACTUARIAL

A) Estimating an Age-to-Age Factor Using Exponential Decay

General Formula

$$ATA_{t+1} = (ATA_t)^{decay_{t-t+1}}$$

An age-to-age factor (*ATA*) beginning at time *t+1* years can be estimated by raising the preceding ATA factor beginning at time *t* years to a selected exponent (*decay*) for the corresponding period.

Example Calculation:

$$ATA_{19-20} = (ATA_{18-19})^{decay_{19-20}}$$

$$ATA_{19-20} = (1.0040)^{0.930}$$

$$ATA_{19-20} = 1.0037$$

Assumptions:

$$ATA_{18-19} = 1.0040$$

$$decay_{19-20} = 0.930$$

B) Estimating an Age-to-Ultimate Factor Using Exponential Decay

General Formula

$$ATU_{t+1} = \exp[\ln(ATA_t)\frac{decay_{t+1-Ult}}{1-decay_{t+1-Ult}}]$$

An age-to-ultimate factor (*ATU*) beginning at time *t+1* years can be estimated by raising the preceding ATA factor beginning at time *t* years to a selected exponent (*decay*) for the corresponding period.

Example Calculation:

$$ATU_{19-Ult} = \exp[\ln(ATA_{18-19})\frac{decay_{19-Ult}}{1-decay_{19-Ult}}]$$

$$ATU_{19-Ult} = \exp[\ln(1.0040)\frac{0.930}{1-0.930}]$$

$$ATU_{19-Ult} = 1.0545$$

Assumptions:

$$ATA_{18-19} = 1.0040$$

$$decay_{19-Ult} = 0.930$$

C) Calculating Exponential Decay Implied in Age-to-Ultimate Factor

General Formula

$$decay_{t-Ult} = \frac{\ln(ATU_t)}{\ln(ATU_t) + \ln(ATA_{t-1})}$$

The implied decay exponent can be calculated given the tail factor at time *t* years and the immediately preceding ATA factor.

Example Calculation:

$$decay_{19-Ult} = \frac{\ln(ATU_{19-Ult})}{\ln(ATU_{19-Ult}) + \ln(ATA_{18-19})}$$

$$decay_{19-Ult} = \frac{\ln(1.0545)}{\ln(1.0545) + \ln(1.0040)}$$

$$decay_{19-Ult} = 0.930$$

Assumptions:

$$ATA_{18-19} = 1.0040$$

$$ATU_{19-Ult} = 1.0545$$

A sample Excel worksheet is available at https://archeractuarial.com/tools/